

Homotopy and the Vietoris-Rips homology

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Abstract: Homotopy theory for topological spaces is generally studied in the context of CW complexes, which are, in particular, spaces which contain an uncountably infinite number of points. The application of topological ideas to data, however, raises the question of how to make homotopical constructions directly on finite sets of points. We discuss here a construction of homotopy groups on point clouds and other combinatorial objects, discuss their basic properties, and illustrate them with several elementary calculations. We then give a new construction of a Vietoris-Rips-type homology theory, show that it is invariant with respect to these homotopies, and that it is isomorphic to the usual Vietoris-Rips homology on finite metric spaces.