The geometry of synchronisation problems and group actions

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A main objective in the analysis of high dimensional data sets is the discovery of the geometry and topology of its underlying space. In recent years much progress has been made in developing methodology that combines insights from Topology, Geometry, and Machine Learning. This course will provide an introduction to some of the recent and exciting developments.

We must remember that although data it typically thought of as numbers, or points in some Euclidean space, it can in fact take many different forms. In this set of lectures, our ultimate goal is to understand how to classify objects with some non-trivial geometry, for example, three-dimensional shapes. We will introduce methods from spectral graph theory and explain how to extend them to deal with the problem of synchronisation, which can be roughly stated as follows. Given a collection of objects, is there a corresponding set of transformations (for instance, orthogonal transformations in the case of shapes in three dimensions) that collects them into orbits? Each orbit of this partially defined action can be regarded as a class of shapes that are similar from the point of view of assumed transformations, and we call them synchronisable. I will discuss the problem of synchronisation from the point of view of the properties of a Laplace operator that can be associated with the data in a very natural way. I will present the necessary geometric and topological machinery that goes into the precise resolution of the problem. Our main motivation will be recent papers by Singer and coworkers, and our approach is based on the recent joint work with Sayan Mukherjee and Tingran Gao. The talks will be accessible to a wide mathematical audience, including posgraduate students.