

Resolving persistent homology

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Persistent homology summarizes geometric objects -- point sets, images, (graphs of) functions, shapes, and so on -- using topology as a recording device. The resulting topological summaries are algebraic gadgets consisting of families of vector spaces indexed by geometric parameters that reflect the "multiscale" nature of the summary. This course introduces algebraic tools to understand what persistent homology is, in a precise mathematical sense, and consequently how to think about topological summaries by drawing pictures of them. This framework applies to the usual setting of one parameter but works as well for multiple parameters, be they discrete or continuous, and much can be learned by exploring analogies between the usual and multiparamter settings. The algebraic concepts introduced include notions of partially ordered set (poset), module, generator (birth), cogenerator (death), presentation, encoding, and decomposition. Motivating examples drawn from published research and work in progress include statistical analyses of brain arteries and fruit fly wings.