

Persistent Homology

5ta ESCUELA DE ANALISIS TOPOLOGICO DE DATOS, November 23

References:

- Sara Kalisnik's lecture notes for an applied topology course at Brown University

1. Given a point cloud X and a radius r , prove that $\check{C}(X, r) \subset VR(X, r) \subset \check{C}(X, 2r)$.
2. Let S be a set of three points in the plane which form an acute triangle (all angles below $\frac{\pi}{2}$), and let T be a set of three points in the plane which form an obtuse triangle (one angle above $\frac{\pi}{2}$).
 - Draw the Voronoi diagrams and Delaunay triangulations of S and T .
 - Draw the family of Alpha-complexes $\text{Alpha}(S, r)$ and $\text{Alpha}(T, r)$, for all radii r (note that there's only a finite number of radii at which these complexes change!). Which family contains a member that is homeomorphic to a circle?
3. Recall that for any finite metric space X and parameter $R > 0$, we define the Vietoris-Rips complex of X with respect to R , $V(X, R)$, to be the abstract simplicial complex with vertex set X , and so that a set $\{x_0, \dots, x_n\}$ is an n -simplex if and only if $d(x_i, x_j) \leq R$ for all $0 \leq i, j \leq n$. Let X be the collections of points on the unit circle $\{(\pm 1, 0), (0, \pm 1), (\pm \frac{\sqrt{2}}{2}, \pm \frac{\sqrt{2}}{2})\}$. These are the points of a regular octagon, on the unit circle. Determine the homology of $V(X, R)$ for all values of R . Also, determine the effect on homology of the inclusion $VR(X, R) \rightarrow VR(X, R')$ whenever $R \leq R'$.